INFOMGP - GAME PHYSICS

EXERCISES LECTURE 1-2-3

EXERCISE 1.1

Suppose you are programming a 2D game in which the main character jumps into a vehicle with constant velocity of 30 km/h in the x-direction. The vehicle is at position $(10\ m\ , 20\ m)$ at frame t. Where will be the vehicle at the next frame if the frame rate is 60 FPS?

We have the equation of movement $p_o(t + \Delta t) = p_o(t) + v\Delta t$
We apply it to our situation (along x) where $p_o(t + \Delta t) = 10 + 30 \times \frac{1000}{3600} \times \frac{1}{60} = 10 + 0.14 = 10.14$
The vehicle will be at the position $(10.14\ m, 20\ m)$.

EXERCISE 1.2

Suppose a NPC police officer steps up a speed trap on a straight section of a road. He times the player who covered 400 meters in 14 seconds. If the speed limit on that section is 90 km/h, will the player have a ticket?

We have the equation of average velocity $\bar{v} = \frac{\Delta p_o}{\Delta t}$
We apply it here where $\bar{v} = \frac{0.4}{14} = 0.029\ km/s = 102.9\ km/h$
The player will receive a ticket.

EXERCISE 1.3

Suppose the player's car is going at 60 km/h and 5 seconds later it is going at 70 km/h? What is its acceleration?

We have the acceleration equation $a = \frac{\Delta v}{\Delta t} = \frac{v(t+\Delta t)-v(t)}{\Delta t}$
We apply it here where $a = \frac{70-60}{5} \times \frac{1}{3.6} = 0.56\ m/s^2$
The acceleration is about 0.56 m/s^2, *i.e.* speeds up by half a m/s every second.

Exercise 1.4

Suppose the player is driving a car and suddenly has to break. He is going at 60 km/h and the car can decelerate at a rate of $-5.55\ m/s^2$. How much time does the player need to stop?

We have the velocity equation $v(t + \Delta t) = v(t) + a\Delta t$
We apply it here where $0 = 60 \times \frac{1}{3.6} + (-5.55) \times t \Leftrightarrow t \approx 3$ seconds
The player stops after about 3 seconds.

Exercise 1.5

Same situation but we want to know if the player will hit a car that is 20 meters in front.

We have the velocity equation $v(t + \Delta t)^2 = v(t)^2 + 2a\Delta p_o$
We apply it here where $0 = (60 \times \frac{1}{3.6})^2 + 2 \times (-5.55) \times \Delta p_o \Leftrightarrow \Delta p_o = 25{,}02\ m > 20\ m$
The player will hit the car.

Exercise 1.6

Suppose a player wants to launch birds from a catapult. The player catapults them at a speed of 20 m/s at a 30 degree angle. How much time will it take a bird to come back down to the ground?

We need to calculate the initial vertical velocity of the bird $v_{vertical} = v \times \sin(angle)$
Here we have $v_{vertical} = 20 \times \sin(30\ deg) = 20 \times 0.5 = 10\ m/s$
We also know the velocity equation $v(t + \Delta t) = v(t) + a\Delta t$
We apply it on the vertical axis to find the time at maximum height:
$0 = 10 + (-9.81) \times t \Leftrightarrow t = 1.02$ seconds
It takes twice the time to get back to the ground, so $t = 2.04$ seconds.
The bird came back to the ground after 2.04 seconds.

Exercise 1.7

Same situation but we want to know how much time it will take to hit a pig 30 meters away.

We first calculate the horizontal velocity $v_{horizontal} = 20 \times \cos(30\ deg) = 17.32$ m/s
Then we can calculate the time needed to travel 30 meters using the equation $v = \frac{\Delta p_o}{\Delta t}$
Here we have $\Delta t = \frac{\Delta p_{o\ horizontal}}{v_{horizontal}} = \frac{30}{17.32} = 1.73$ seconds
The bird will hit the pig after 1.73 seconds.

Exercise 1.8

Same situation but we want to know how far away the bird will hit the ground.

We know the velocity equation $v = \frac{\Delta p_o}{\Delta t}$
So we have here (see exercises 1.7 and 1.8):
$\Delta p_{o\ horizontal} = v_{horizontal} \times \Delta t = 17.32 \times 2.04 = 35.33$ meters
The bird hits the ground 35.33 meters away from the launch site.

Exercise 1.9

In a platform game, the player can run off an edge to land on a lower platform. If the player runs at 10 m/s from an 8 meter high platform, how far away will he land on the ground?

The vertical displacement is given by $\Delta p_{o\ vertical} = v_{vertical} \times \Delta t + \frac{1}{2} a_{vertical} \Delta t^2$
So we get: $-8 = 0 \times \Delta t + \frac{1}{2} \times (-9.81) \times \Delta t^2 \Leftrightarrow \Delta t^2 = 1.63$, and so $\Delta t = 1.28$ seconds
It takes 1.28 seconds for the player to hit the ground.
We also know that $v_{horizontal} = \frac{\Delta p_{o_{horizontal}}}{\Delta t}$, where here $\Delta p_{o_{horizontal}} = 10 \times 1.28 = 12.8$ meters
The player lands 12.8 meters away from the edge of the platform.

Exercise 1.10

Suppose a ball weighting 1.5 kg is rolling down a ramp which is positioned at a 30 degree incline. What it the normal force of the ramp?

We have the normal force $F_N = W \times \cos(angle)$
We apply it here where $F_N = 1.5 \times 9.81 \times \cos(30\ deg) = 12.74$ Newtons
The normal force is 12.74 Newtons.

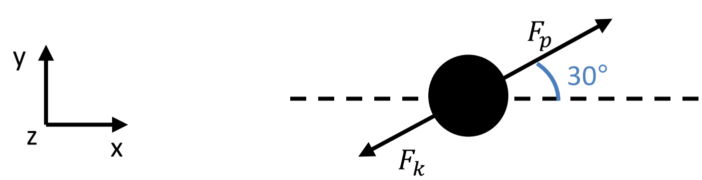
EXERCISE 1.11

Suppose you are programming a racing game with a car weighting 1 T. What are the static and kinetic friction forces between the rubber tires and the dry concrete flat road?

We first need to calculate the weight of the car $W = m \times g = 1000 \times 9.81 = 9810$ Newtons
As the road is flat the normal force is also 9810 Newtons but in the opposite direction.
The coefficient of static friction is 1.0, so $F_s = 9810$ Newtons.
The coefficient of kinetic friction is 0.9, so $F_k = 8829$ Newtons.

EXERCISE 1.12

Suppose a game character is pulling a ball across a field. Initially the ball is sitting still at the origin $(0, 0)$ and its mass is 5 kg. The player pulls with a constant force of 400 N at an angle of 30 degrees with the x-direction of the field. The coefficient of kinetic friction is 0.25. Where the ball will be after respectively 1, 2 and 3 seconds?



We can calculate F_k from the weight and the normal force:

$W = -5 \times 9.81 = -49.05$ N (oriented in the - z-direction)
$F_N = 49.05$ N (oriented in the z-direction)
$F_k = \mu_k F_N = 0.25 \times 49.05 = 12.26$ N (opposing the direction of F_p)

Next we calculate the net force on the ball (F_N and W cancel each other):

$$F_p = 400 \times \begin{pmatrix} \cos(30\ deg) \\ \sin(30\ deg) \end{pmatrix} = \begin{pmatrix} 346.41 \\ 200 \end{pmatrix}$$
$$F_k = 12.26 \times \begin{pmatrix} \cos(180 + 30\ deg) \\ \sin(180 + 30\ deg) \end{pmatrix} = \begin{pmatrix} -10.62 \\ -6.13 \end{pmatrix}$$

$$F_{net} = F_p + F_k = \begin{pmatrix} 335.79 \\ 193.87 \end{pmatrix}$$

From that we can calculate the acceleration of the ball:

$$F_{net} = m \times a$$

$$\begin{pmatrix} 335.79 \\ 193.87 \end{pmatrix} = 5 \times \begin{pmatrix} a_x \\ a_y \end{pmatrix} \Leftrightarrow a = \begin{pmatrix} 67.16 \\ 38.77 \end{pmatrix}$$

Then we integrate the acceleration to calculate the position:

$\Delta p_o = v(t) \times \Delta t + \frac{1}{2} \times a \times \Delta t^2$, but as $v(0) = 0$ we have $\Delta p_o = \frac{1}{2} \times a \times \Delta t^2$, which is here

$$\Delta p_o = \frac{1}{2} \times \begin{pmatrix} 67.16 \\ 38.77 \end{pmatrix} \times \Delta t^2$$

Finally we can calculate the position at a specific time:

At $t = 1$ s : $p_o(1) = \frac{1}{2} \times \begin{pmatrix} 67.16 \\ 38.77 \end{pmatrix} \times 1^2 = \begin{pmatrix} 33.58 \\ 19.38 \end{pmatrix}$

At $t = 2$ s : $p_o(2) = \frac{1}{2} \times \begin{pmatrix} 67.16 \\ 38.77 \end{pmatrix} \times 2^2 = \begin{pmatrix} 134.32 \\ 77.54 \end{pmatrix}$

At $t = 3$ s : $p_o(3) = \frac{1}{2} \times \begin{pmatrix} 67.16 \\ 38.77 \end{pmatrix} \times 3^2 = \begin{pmatrix} 302.22 \\ 174.46 \end{pmatrix}$

EXERCISE 1.13

Imagine the player can shoot cannon balls to his opponent. At some point on its trajectory the ball speed is 100 km/h (that's high) and the ball has a radius of 10 cm. The density of air is 1.2 kg/m^3 and the drag coefficient of the ball is 0.47. What is the drag force opposing the motion of the ball?

We have the high velocity drag force equation $F_{D_{high}} = -\frac{1}{2} \times \rho \times v^2 \times C_d \times A$

Here we have the reference area $A = \pi r^2 = \pi \times 0.1^2$

So we have the force $F_{D_{high}} = -\frac{1}{2} \times 1.2 \times \left(100 \times \frac{1}{3.6}\right)^2 \times 0.47 \times \pi \times 0.1^2 = -6.8$ Newtons

The drag force is 6.8 Newtons in the direction opposing the motion.

EXERCISE 1.14

Suppose a game character of volume 0.1 m^3 and weighting 80 kg. What density of fluid is necessary for the character to float?

The character floats if its weight is smaller than the buoyancy, so if $W < F_B$

We know that the weight of the character is $m \times g$ and the buoyancy is $\rho \times g \times V$.

So the character floats if $m \times g < \rho \times g \times V \Leftrightarrow m < \rho \times V \Leftrightarrow 80 < \rho \times 0.1 \Leftrightarrow \rho > 800\ kg/m^3$

The character floats if the density is larger than 800 kg/m^3, *e.g.* water but not air.

EXERCISE 1.15

Suppose a game character throws a ball with a force of 400 N on a distance of 0.75 meter. The ball has a mass of 145 g. What speed will the ball have when the character lets go of it?

We have the work-energy theorem $F\Delta p_o = \frac{1}{2} \times m \times (v(t+\Delta t)^2 - v(t)^2)$
We apply it here where $400 \times 0.75 = \frac{1}{2} \times 0.145 \times (v(t+\Delta t)^2 - 0) \Leftrightarrow v(t+\Delta t) = 64.33$ m/s
The speed of the ball is 64.33 m/s.

EXERCISE 1.16

In a roller coaster game, the cart starts at a height of 100 meters and goes down at the bottom of the first slope. The mass of the cart and the player in it is 200 kg. During the descent, 2000 Joules of energy are lost in heat and sound. How fast the cart goes at the bottom of the slope?

We know that the energy is conserved, so $E_K(t+\Delta t) + E_P(t+\Delta t) = E_K(t) + E_P(t) + E_o$
We apply it here where $\frac{1}{2} mv(t+\Delta t)^2 + mgh(t+\Delta t) = \frac{1}{2} mv(t)^2 + mgh(t) + E_o$
$\Leftrightarrow \frac{1}{2} \times 200 \times v(t+\Delta t)^2 + 200 \times 9.81 \times 0 = \frac{1}{2} \times 200 \times 0 + 200 \times 9.81 \times 100 + 2000$
$\Leftrightarrow v(t+\Delta t) = 44.52$ m/s
The cart's speed is 44.52 m/s at the bottom of the slope.

EXERCISE 1.17

In a golf game, the player defines the impulse force applied to the ball. Suppose the mass of the ball is 45 g and the player input an impulse of $\binom{3}{2}$. What is the ball velocity?

We know the momentum/impulse equation $\Delta p = F\Delta t = m \times (v(t+\Delta t) - v(t))$
We apply it here where $\binom{3}{2} = 0.045 \times \left(\binom{v_x}{v_y} - \binom{0}{0}\right) \Leftrightarrow \binom{v_x}{v_y} = \binom{66.7}{44.4}$
The resulting velocity is $\binom{66.7}{44.4}$.

EXERCISE 1.18

Suppose you want to program a game based on the Wheel of Fortune game. If the player gives the wheel an initial angular velocity of 8 rad/s, and the pegs decelerate it at a rate of 2 rad/s^2, what is its angular displacement after 3 seconds?

We have the angular displacement equation $\Delta\theta = \omega(t)\Delta t + \frac{1}{2}\alpha\Delta t^2$
We apply it here where $\Delta\theta = 8 \times 3 + 0.5 \times (-2) \times 3^2 = 15$ rad
The wheel has rotated by 15 radians (i.e. nearly 2 and a half full rotations).

EXERCISE 1.19

Same situation but it takes 10 seconds to the wheel to stop. The wheel has a 3 meter radius. What is the wheel's tangential acceleration?

We first calculate the angular acceleration:

$$\omega(t + \Delta t) = \omega(t) + \alpha\Delta t$$
$$0 = 8 + 10\alpha \Leftrightarrow \alpha = -0.8 \text{ rad/s}^2$$

We can then find the tangential acceleration:

$$a_t = \alpha \times r = -0.8 \times 3 = -2.4\, m/s^2$$

The tangential acceleration is $-2.4\, m/s^2$.

EXERCISE 1.20

In a racing game the player's car get hit by an opponent. The mass of the player's car is 1.2 T, the impact is at a distance of 3 meters from the COM in a direction perpendicular to the motion. The effect is that the player's car rotates with an angular acceleration of 0.5 rad/s^2. What is the torque of the collision?

We have the torque definition $\tau = m \times r^2 \times \alpha$
We apply it here where $\tau = 1200 \times 3^2 \times 0.5 = 5400\, Nm$
The torque is 5400 Nm.

EXERCISE 1.21

Imagine a blue hedgehog rolling down a 10 meter high hill with a mass m. Its inertia is given by $\frac{2}{5}mr^2$. What is its translational speed at the bottom of the hill?

We know that the energy is conserved, so

$E_{Kt}(t + \Delta t) + E_P(t + \Delta t) + E_{Kr}(t + \Delta t) = E_{Kt}(t) + E_P(t) + E_{Kr}(t)$

$\Leftrightarrow \frac{1}{2}mv(t + \Delta t)^2 + mgh(t + \Delta t) + \frac{1}{2}I\omega(t + \Delta t)^2 = \frac{1}{2}mv(t)^2 + mgh(t) + \frac{1}{2}I\omega(t + \Delta t)^2$

Here we have

$\frac{1}{2}mv(t + \Delta t)^2 + 0 + \frac{1}{2}I\omega(t + \Delta t)^2 = 0 + 10mg + 0$

$\Leftrightarrow \frac{1}{2}mv(t + \Delta t)^2 + \frac{1}{2}\frac{2}{5}mr^2\omega(t + \Delta t)^2 = 10 \times m \times 9.81$

$\Leftrightarrow 0.5v(t + \Delta t)^2 + 0.2r^2\omega(t + \Delta t)^2 = 98.1$

We know that, if the object rolls without slipping then $v = v_t = r \times \omega$, so $v^2 = r^2 \times \omega^2$, so

$0.5v(t + \Delta t)^2 + 0.2v(t + \Delta t)^2 = 98.1$

$\Leftrightarrow 0.7v(t + \Delta t)^2 = 98.1$

$\Leftrightarrow v(t + \Delta t) = 11.84$ m/s

The linear velocity at the bottom of the hill is 11.84 m/s.